# LIFE AND RELIABILITY OF ROTATING DISKS

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#### INTRODUCTION

In aerospace applications, an engineer must be especially cognizant of size and weight constraints which affect design decisions. Although designing at or below the material fatigue limit may be desirable in most industrial applications, in aerospace applications it is almost mandatory to design certain components for a finite life at an acceptable probability of survival. Zaretsky (1987) outlined such a methodology based in part on the work of W. Weibull (1939, 1951) and G. Lundberg and A. Palmgren (1947a, 1947b, 1952). Zaretsky's method (1987) is similar in approach to that of Ioannides and Harris (1985).

Mahorter et al. (1985) discuss the accuracy of life prediction techniques for military turbine engine components such as compressor and turbine disks. The development of a 0.794 mm (0.03 in.) crack in any of the critical areas of a disk, such as bolt holes, bore, dovetail, etc., is considered the end of its low cycle fatigue (LCF) life. This is the life,  $L_{0.1}$ , at which the military requires a 99.9 percent probability of survival or a 0.1 percent failure rate. Disk retirement policy requires that the disks be removed from service or reworked at their  $L_{0.1}$  life. The statistically predicted lives for bolt-hole cracks were shorter than the deterministically predicted values. These results imply that a deterministic approach to life prediction is not necessarily conservative, and that a probabilistic approach is viable in light of all the statistical variations in the design parameters.

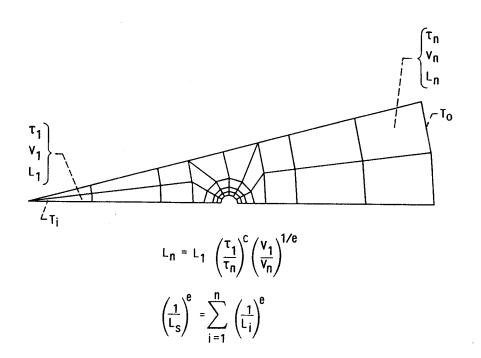
In view of the aforementioned, it is the objective of this work to (a) apply the method of Zaretsky (1987) to statistically predict the life of a generic solid disk with and without bolt holes, (b) determine the effect of disk design variables, thermal loads, and speed on relative life, and (c) develop a generalized equation for determining disk life by incorporating only these variables.

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### **OVERVIEW**

### FINITE ELEMENT MODEL

Parametric analytical studies were conducted to investigate the effect of varying physical dimensions and speed on the relative lives of a generic solid disk. The physical model of the solid disk requires that the structure be divided into a series of components (elements). This is most easily done by considering the disk as a collection of concentric rings. The disk was modeled with 10 rings of equal radial increment for the parametric values of disk diameter studied.



## GOVERNING EQUATIONS

The following equation for the 10-percent life of a uniform solid rotating disk is obtained:

$$L_{10} = A \left[ \left( \frac{K_D}{D} \right)^{20} \left( \frac{K_t}{t} \right) \left( \frac{K_N}{N} \right)^{14.3} \left( \frac{9}{c} \right)^{0.606} \left( \frac{K_T}{\Delta T} \right)^{0.52} \right] \exp(K_L \tau_L)$$

Introducing the concept of a Dynamic Speed Capacity  $N_0$ , which is defined by Zaretsky (1987) as the speed that would produce a theoretical life of one million stress cycles,

$$N_{o} = Ax10^{-6} \left[ \left( \frac{K_{D}}{D} \right)^{20} \left( \frac{K_{t}}{t} \right) K_{N}^{14.3} \left( \frac{9}{c} \right)^{0.606} \left( \frac{K_{T}}{\Delta T} \right)^{0.52} \right] exp(K_{L}\tau_{L})$$
 1/14.3

and for any speed N for a given disk geometry.

$$L_{10} = \left(\frac{N_o}{N}\right)^{14.3} \times 10^6$$

### SYMBOLS

material constant, stress cycles Α С stress-life exponent D disk diameter, m (in.) Ε material Young's modulus, N/m<sup>2</sup> (psi) Weibull slope or modules е proportionality constant, m (in.)  $K_{\mathbf{D}}$ proportionality constant,  $m^2/N$  (psi<sup>-1</sup>) KŢ. proportionality constant, rpm  $K_N$  $K_{\mathbf{T}}$ proportionality constant, K (°F) proportionality constant, m (in.) Κt L life, stress cycles  $L_{\mathbf{S}}$ system life, stress cycles ten-percent life, life at which 90 percent of a population survive,  $L_{10}$ stress cycles N speed, rpm dynamic speed capacity, rpm  $N_{O}$ Т temperature, K (°F) t disk thickness, m (in.) stress volume, m<sup>3</sup> (in.<sup>3</sup>)  $\Delta T$ temperature difference, K (°F) shear stress, N/m<sup>2</sup> (psi) τ fatigue endurance limit, N/m<sup>2</sup> (psi) τī. Subscripts:  $i\frac{\text{th}}{}$  component, and denotes inner disk radius i

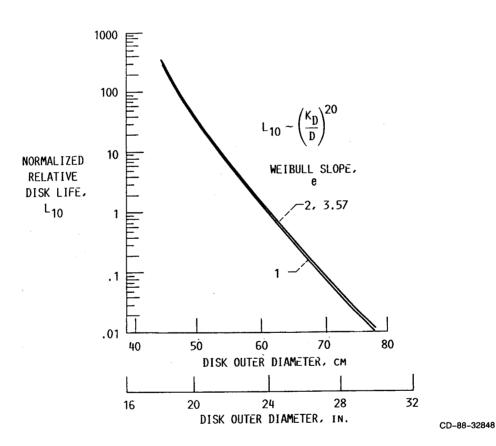
reference point, and denotes outer disk radius

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## POSTER PRESENTATION:

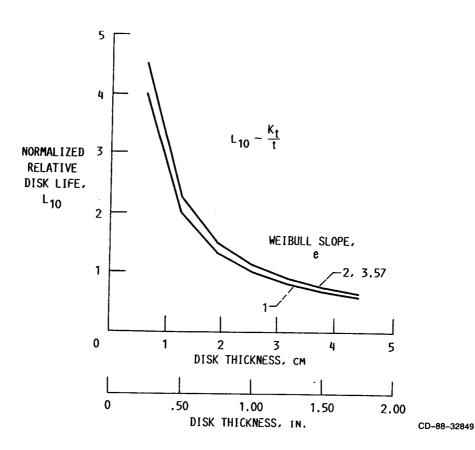
## EFFECT OF DISK DIAMETER

The effect of disk diameter on life for a speed of 9000 rpm and a stress-life exponent, c, of 9 is shown. As the disk diameter is increased, both the stress and the stress volume increase and life will decrease. The effect of Weibull slope is negligible. D is the disk diameter,  $K_{\rm D}$  is a constant equal to 0.61 m (24.0 in.), and  $L_{10}$  is the life at a 90-percent probability of survival (or the life where 10 percent of the disks have failed).



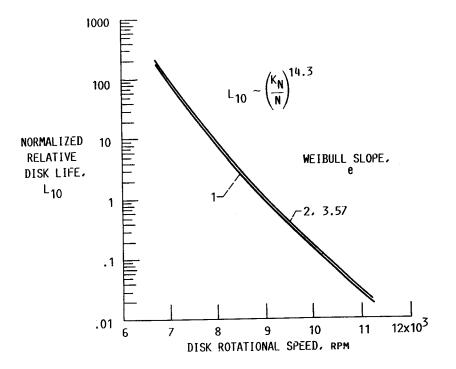
## EFFECT OF DISK THICKNESS

The effect of disk thickness on the  $L_{10}$  life is shown. Because the stressed volume is increased, life will decrease. The effect of Weibull slope is negligible. t is the disk thickness, and  $K_{t}$  is a constant equal to 0.0254 m (1.0 in.).



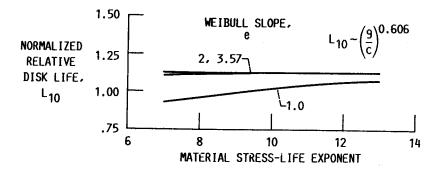
## EFFECT OF SPEED

As disk speed is increased, stresses within the disk will increase. These stress increases will cause a decrease in life. The effect of disk rotational speed on  $\,L_{10}\,$  life is shown. N is the disk speed and  $\,K_N\,$  is a speed constant equal to 9000 rpm.



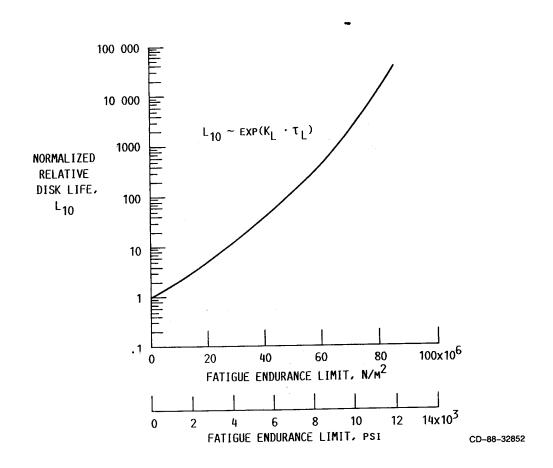
# EFFECT OF STRESS-LIFE EXPONENT

For each stressed elemental volume within the body of the disk, the life was determined by using an inverse stress-life relation. Not all materials will exhibit the same stress-life relation. The stress-life exponent is generally determined experimentally. For the previous calculations, a stress-life exponent of 9 was assumed. Using varying values of c with a reference disk, the effect of the stress-life exponent on the  $L_{10}$  life of the disk was determined. These results are shown.



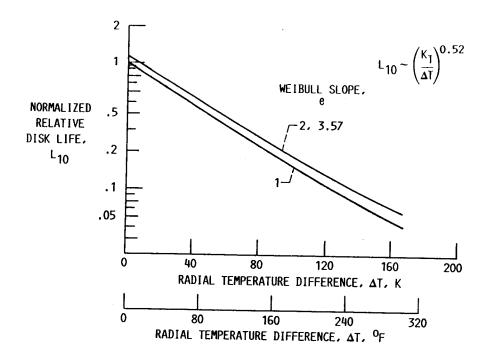
## EFFECT OF ENDURANCE LIMIT

A concept first espoused by Ioannides and Harris (1985) was the application of a fatigue limit in the determination of the lives of the elemental stressed volumes. Basically, the concept is that where the shearing stress is equal to or less than the value determined or assumed for the fatigue limit, the probability of survival for that elemental stressed volume is 100 percent. The results of assuming varying values of a fatigue limit for the shearing stress is shown where  $\tau_L$  is the fatigue endurance limit at or below which stress no failure is expected to occur and  $K_L$  is equal to  $4.02 \times 10^{-8}$  m<sup>2</sup>/N (2.77x10<sup>-4</sup> psi<sup>-1</sup>).



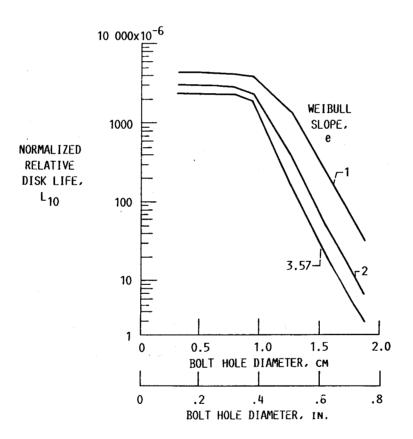
# EFFECT OF TEMPERATURE GRADIENTS

The effect of a radial temperature difference upon the overall disk relative life was determined by considering the thermal stresses superimposed upon the centrifugal disk stresses due to disk rotation. A steady-state radial temperature distribution was applied to a disk with a very small central hole. A disk with a small central hole was considered because this configuration relieved a mathematical singularity which occurs for isothermal boundary conditions upon a solid disk. The effect of various uniform radial linear temperature gradients on disk relative life is shown.  $\Delta T$  is the total radial temperature difference and  $K_T$  is a constant equal to 0.56 K when  $\Delta T$  is in Kelvin or 1F when  $\Delta T$  is in Fahrenheit, and where  $\Delta T$  is equal to or greater than  $K_T$ .



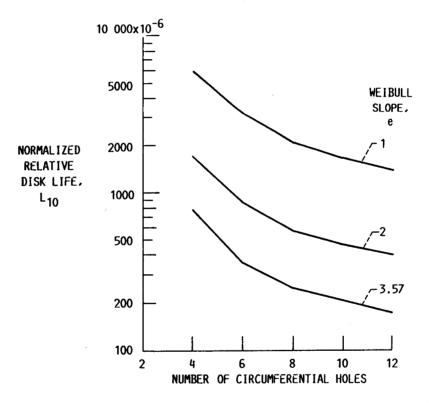
### EFFECT OF HOLE SIZE

As discussed by Mahorter et al. (1985), tie-bolt holes will be the critical location for failure in a disk. Using the finite element model, the effect on disk life of bolt-hole size, location, and number was determined. The effect of bolt diameter on disk fatigue life is shown. It would appear that at bolt holes having a diameter of less than 10.2 mm (0.4 in.), the effect of hole size is nominal. At bolt holes larger in diameter, the effect is most significant. This analysis would suggest that the bolt holes in a disk should be smaller but more numerous.



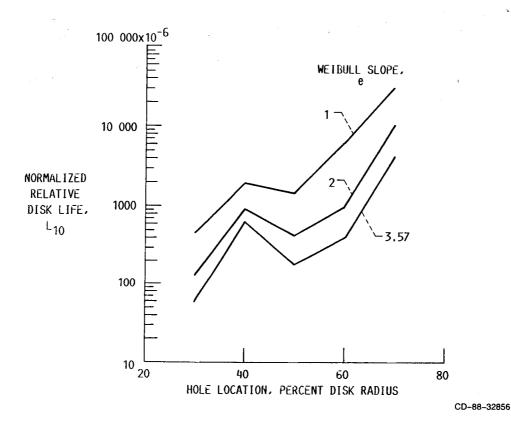
## EFFECT OF HOLE NUMBER

The effect of increasing the number of bolt holes is shown. Increasing the number of holes and keeping the hole diameter less that 10.2 mm (0.4 in.) appear to have less of an effect on life than having fewer holes with a larger diameter.



### EFFECT OF HOLE LOCATION

The effect of hole location in the disk is shown as a percent of the disk radius measured from the axis of rotation. The results indicate a general trend of increasing life as the holes are moved radially outward. This would not be unexpected because stresses decrease with increasing distance from the center. The trend is not totally convincing, however, since there is an inversion of the trend at a location between 40 and 50 percent of the disk radius; that is, life decreases and then begins to increase again. It cannot be determined with reasonable certainty that this is correct or whether the finite element mesh size was properly selected for these calculations. Further analysis is required. However, it can be reasonably concluded that the bolt holes should be placed as far from the center of the disk as is practical.



#### REFERENCES

- Ioannides, E., and Harris, T.A., 1985, "A New Fatigue Life Model for Rolling Bearings," Journal of Tribology, Vol. 107, No. 3, pp. 367-378.
- Lundberg, G., and Palmgren, A., 1947a, "Dynamic Capacity of Rolling Bearings," Ingeniors Vetanskaps Akadmien-Handlinger, No. 196.
- Lundberg, G., and Palmgren, A., 1947b, "Dynamic Capacity of Rolling Bearings," Acta Polytechnica, Mechanical Engineering Series, Vol. 1, No. 3, pp. 1-50.
- Lundberg, G., and Palmgren, A., 1952, "Dynamic Capacity of Rolling Bearings," Acta Polytechnica, Mechanical Engineering Series, Vol. 2, No. 4.
- Mahorter, R., London, G., Fowler, S., and Salvino, J., 1985, "Life Prediction Methodology for Aircraft Gas Turbine Engine Disks," AIAA Paper 85-1141.
- Weibull, W., 1939, "The Phenomenon of Rupture in Solids," <u>Ingeniors Vetanskaps</u>
  Akademien-Handlinger, No. 153.
- Weibull, W., 1951, "A Statistical Distribution Function of Wide Applicability," Journal of Applied Mechanics, Vol. 18, No. 3, pp. 293-297.
- Zaretsky, E.V., 1987, "Fatigue Criterion to System Design, Life and Reliability," <u>Journal of Propulsion and Power</u>, Vol. 3, No. 1, pp. 76-83.